Supplementary Appendix to:

Convicts and Comrades

by Hazal Sezer

H Model with Elite and Regular Firms

In this section, I expand the general equilibrium framework to analyze the labor market impacts of convict labor. I develop a model with two firms: one using convict labor (elite firm) and one not using it (regular firm). This model highlights labor reallocation and shows that regular firms also benefit from the policy. Elite firms, connected politically, gain access to low-cost convict labor, influencing labor market dynamics and potentially lowering wages for free workers.[1] The model, reflecting multiple firms in a county, indicates that convict labor leads to lower wages for free workers, reallocates labor from elite to regular firms, and increases union and strike activities. This framework is applicable to general firm–type distribution and the free workers' endogenous labor supply.

H.1 Convict and Free Labor

This paper explores a period in the Southern U.S. when oppressive laws caused Black individuals to make up most convict labor while white individuals dominated free labor. These coercive laws exploited imprisoned Black individuals as cheap labor, segregating labor groups and prohibiting unionization among convict laborers (Hiller, 1914).

Convict labor was a limited resource—every convict in prison contributed labor (U.S. Bureau of Labor, 1886). This capacity constraint meant any increase in prison capacity could significantly impact free labor wages and stimulate union formation. The model includes this constraint to understand convict labor employment. Convict laborers did not receive wages ($w_c = 0$), while free laborers were paid ($w > 0$).

H.2 Elite and Regular Firms

There are two types of firms: elite and regular. Elite firms employ both types of workers, convict and free labor, while regular firms only hire free labor. Both types of firms produce a homogeneous product without quality differentiation and compete in the labor market.

[1] This hints that elite firms might use their political connections to influence labor markets and depress wages for free workers, independent of convict labor. However, declining wages and increased unionization after convict labor's introduction suggest these firms hadn't used their monopsony power before.

The elite firm's (e) production function is:

$$y_e = z_e(\theta l_{c,e} + l_{f,e})^\alpha k_e^\beta, \tag{H.1}$$

where y_e is the output of the elite firm and $\alpha + \beta < 1$. The elite firm hires $l_{c,e}$ units of convict labor and $l_{f,e}$ units of free labor and acquires capital in the measure of k_e. The total factor productivity of the elite firm is denoted by z_e. The model permits the potential for productivity disparity between free and convict laborers through the θ multiplier preceding $l_{c,e}$. The elite firm independently determines the quantity of each labor type to employ and the capital to procure. The number of convict laborers hired, $l_{c,e}$, is subject to a capacity constraint $l_{c,e} \leq \bar{l}$. As the wage rate for convict labor is zero, the elite firm's convict labor capacity constraint always binds, i.e., $l_{c,e} = \bar{l}$. I assume $w_c = 0$ without loss of generality. If $w_c = c$, the convict labor capacity constraint of the elite firm still binds as long as c is sufficiently small. In this case, the total labor cost of the elite firm would become $c\bar{l}$, and the optimization program would remain unaltered since $c\bar{l}$ is a constant lump sum. The Bureau of Labor reports (U.S. Bureau of Labor, 1886, 1925) implies that the unit convict labor costs (w_c) were sufficiently small by that time, such that convict labor supply was employed in production to its full extent.

The regular firm's (r) production function is:

$$y_r = z_r l_{f,r}^\alpha k_r^\beta \tag{H.2}$$

where y_r is the output of the regular firm and $\alpha + \beta < 1$. The regular firm hires $l_{f,r}$ units of free labor and acquires capital in the measure of k_r. The total factor productivity of the regular firm is denoted by z_r.

Wages of free labor, w, will be determined endogenously in general equilibrium. Rental rate $r > 0$ of capital is exogenous, with full depreciation of capital. Solving the maximization problem of the elite firm, demands for $l_{f,e}$ and k_e become:

$$l_{f,e} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} - \theta\bar{l} \tag{H.3}$$

$$k_e = \left(\frac{r}{\beta}\right)^{\frac{\alpha-1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} \tag{H.4}$$

By solving the optimization problem for the regular firm, the demands for $l_{f,r}$ and k_r are derived as:

$$l_{f,r} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \tag{H.5}$$

$$k_r = \left(\frac{r}{\beta}\right)^{\frac{\alpha-1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \tag{H.6}$$

H.3 Households and General Equilibrium Wages

There are N households in the economy whose total labor supply equals $\bar{L}$, and they inelastically supply their labor services for the elite and regular firms. The total demand for free labor is expressed as $l_{f,e} + l_{f,r}$. The general equilibrium of the economy is determined by:

$$\bar{L} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right) - \theta\bar{l}, \tag{H.7}$$

which characterizes the equilibrium wage:

$$w = \alpha \left(\frac{\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)}{\bar{L} + \theta\bar{l}} \right)^{\frac{1-\alpha-\beta}{1-\beta}}. \tag{H.8}$$

Equation (H.8) shows that an increase in the capacity constraint for convict labor results in a corresponding decrease in the wages of free labor $\left(\frac{\partial w}{\partial \bar{l}} < 0\right)$. Furthermore, when $\bar{l}$ is sufficiently large, the productivity of convict labor and its capacity become strategic complements. As a result, enhancing convict labor productivity further depresses equilibrium wage level $\left(\frac{\partial^2 w}{\partial \bar{l} \partial \theta} > 0\right)$. This leads to the following proposition:

Proposition I1: *Increasing the capacity constraint of convict labor reduces the equilibrium wage rate. Furthermore, if $\bar{l}$ exceeds a threshold $\bar{l}^*$, an increase in θ further lowers the equilibrium wage rate.*

H.4 Labor Reallocation

A direct implication of equations (H.4) and (H.6) is that a decline in wages is associated with an increased demand for capital across all firms. However, diminishing wages for free labor and elite firms employing convict labor reallocates free laborers from elite to regular firms. Based on this, I formalize the following:

Proposition I2: *Increasing the capacity constraint for convict labor causes a reallocation of free labor from the elite to the regular firm.*

The labor reallocation dynamic in my model parallels Hubmer and Restrepo (2022), where larger firms automate more, reducing the overall labor share, and workers migrate to median firms. In my model, elite firms replace free labor with convict labor, causing free labor to shift to regular firms and increasing their physical labor intensity as wages decline. The key distinction is that Hubmer and Restrepo (2022) focus on substituting labor with capital, while my model emphasizes substituting free labor with convict labor.

H.5 Equilibrium Profits

As the capacity for convict labor expands and equilibrium wages decrease, it is reasonable to expect a corresponding impact on firms' profits. The profits of the elite firm are given by:

$$\Pi_e = \left(\bar{L} + \theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right) + \alpha\theta\bar{l}\left(\bar{L} + \theta\bar{l}\right)^{\frac{1-\alpha-\beta}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}}. \tag{H.9}$$

The capacity constraint of convict labor positively influences the equilibrium profits of the elite firm, i.e., $\frac{\partial \Pi_e}{\partial \bar{l}} > 0$, because the elite firm can access convict labor input at no cost.

The profits of the regular firm are given by:

$$\Pi_r = \left(\bar{L} + \theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right). \tag{H.10}$$

The equilibrium profits of the regular firm increase with the capacity constraint of convict labor, i.e., $\frac{\partial \Pi_r}{\partial \bar{l}} > 0$. This occurs as free labor shifts from the elite firm to the regular one due to the elite's advantage of accessing convict labor without incurring any costs. This gives rise to the ensuing proposition:

Proposition I3: *An increase in the capacity for convict labor increases profits for both elite and regular firms.*

My model shows that convict labor increased firm profits, explaining its high demand and support from both elite and regular firm owners. It suggests regular firms benefited from reduced free labor wages and the reallocation of free labor from elite firms.

H.6 Capital–Labor Cost Ratio

Next, I explore the effects of convict labor on the capital–labor cost ratio, assessing its impact across the entire county, encompassing elite and regular firms. The capital-to-labor cost ratio for the county is denoted as $CI \equiv \frac{rk_e + rk_r}{w_f \gamma \bar{L} + w_c \bar{l}}$. Since $w_c = 0$, the ratio simplifies to:

$$CI \equiv \frac{rk_e + rk_r}{w\bar{L}}. \tag{H.11}$$

The capital-to-labor cost ratio rises with the capacity constraint of convict labor, as both k_e and k_r are increasing in $\bar{l}$, and w is decreasing in $\bar{l}$. The underlying rationale is that increased capital demand by all firms in a county raises total capital costs, while fixed free labor supply and falling wages lower overall labor costs. Drawing from this, I formalize:

Proposition EI: *A rise in convict labor capacity increases the capital-to-labor cost ratio throughout the region.*

H.7 Union Membership and Strikes

Suppose N households, with a total inelastic labor supply of $\bar{L}$, invest J to form a union. This union allows elite firm free laborers to strike against convict labor at the start of production, supported by regular firm free laborers whose wages also fall due to convict labor. The strike could force the elite firm to reduce convict labor to zero. If the strike succeeds, the employee's benefit (Ω) is derived. The equilibrium wage depends on the capacity constraint, denoted as $w(l_c)$:

$$\Omega = \left(w(l_c = 0) - w(l_c = \bar{l})\right)\bar{L}, \tag{H.12}$$

This value determines the free laborer's willingness to pay the unionization cost J. From equation (H.12), as $\bar{l}$ increases, the likelihood of joining the union rises. This is because the wage gap, $w(l_c = 0) - w(l_c = \bar{l})$, increases with $\bar{l}$, consistent with earlier findings.

Free labor will form a union and pay cost J if they believe strikes can succeed. If the elite firm owner can prevent a strike with probability $1 - p$ and a strike succeeds with probability p, convict labor employment at the elite firm is eliminated. If the strike fails, laborers incur a utility loss, δ. Thus, free laborers will join the union and strike if and only if:

$$J < p\Omega - (1-p)\delta = p\left(w(l_c = 0) - w(l_c = \bar{l})\right)\bar{L} - (1-p)\delta. \tag{H.13}$$

As the RHS of equation (H.13) increases with $\bar{l}$, a higher capacity for convict labor directly promotes more unionization and strike activities. This leads to the following proposition:

Proposition I5: *An increase in the capacity for convict labor results in heightened unionization and strike actions.*

The growth in the number of union assemblies is embodied in the probability of unionization within this model. Effects on both extensive and intensive margins are captured in the J term. Convict labor reduces free labor wages while increasing their unionization and strikes. When $\bar{l}$ is large, convict labor productivity and capacity act as strategic complements. Higher convict productivity further lowers equilibrium wages. Thus, increased convict labor productivity amplifies the impact of convict labor capacity on unionization, as shown in the right-hand side of equation (H.13).

I Model Solution and Proofs: Elite and Regular Firms

I.1 Labor Demand and Capital

Elite firm $l_{f,e}$ and k_e: Elite firm's production function is $y_e = z_e \left(\theta l_{c,e} + l_{f,e}\right)^{\alpha} k_e^{\beta}$, $\alpha+\beta < 1$. Solving the maximization problem of the elite firm and taking the first-order conditions:

$$\max_{\{l_{f,e},k_e\}} \Pi_e = y_e - wl_{f,e} - rk_e$$

$$\max_{\{l_{f,e},k_e\}} \Pi_e = z_e \left(\theta\bar{l} + l_{f,e}\right)^{\alpha} k_e^{\beta} - wl_{f,e} - rk_e, \ \ \bar{l} = l_{c,e}$$

Taking the first-order condition w.r.t. $l_{f,e}$:

$$\frac{\partial \Pi_e}{\partial l_{f,e}} \Rightarrow \alpha z_e k_e^{\beta} \left(\theta\bar{l} + l_{f,e}\right)^{\alpha-1} - w = 0$$

$$\theta\bar{l} + l_{f,e} = \left(\frac{w}{\alpha z_e k_e^{\beta}}\right)^{\frac{1}{\alpha-1}}$$

Simplifying $l_{f,e}$ becomes:

$$l_{f,e} = \left(\frac{w}{\alpha z_e k_e^{\beta}}\right)^{\frac{1}{\alpha-1}} - \theta\bar{l}$$

Taking the first-order condition w.r.t. k_e:

$$\frac{\partial \Pi_e}{\partial k_e} \Rightarrow \beta z_e(\theta\bar{l} + l_{f,e})^{\alpha} k_e^{\beta-1} - r = 0$$

Simplifying k_e becomes:

$$k_e = \left(\frac{r}{\beta z_e(\theta\bar{l} + l_{f,e})^{\alpha}}\right)^{\frac{1}{\beta-1}}$$

Plugging k_e into $l_{f,e}$:

$$l_{f,e} = \left(\frac{w}{\alpha z_e \left(\frac{r}{\beta z_e(\theta\bar{l}+l_{f,e})^{\alpha}}\right)^{\frac{\beta}{\beta-1}}}\right)^{\frac{1}{\alpha-1}} - \theta\bar{l}$$

$$l_{f,e} = \left(\frac{w}{\alpha}\left(\frac{\beta}{r}\right)^{\frac{\beta}{\beta-1}} z_e^{\frac{1}{\beta-1}} \left(\theta\bar{l} + l_{f,e}\right)^{\frac{\alpha\beta}{\beta-1}}\right)^{\frac{1}{\alpha-1}} - \theta\bar{l}$$

$$l_{f,e} = \left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{(\beta-1)(\alpha-1)}} z_e^{\frac{1}{(\beta-1)(\alpha-1)}} \left(\theta\bar{l} + l_{f,e}\right)^{\frac{\alpha\beta}{(\beta-1)(\alpha-1)}} - \theta\bar{l}$$

$$\left(l_{f,e} + \theta\bar{l}\right)^{\frac{1-\alpha-\beta}{(\beta-1)(\alpha-1)}} = \left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{(\beta-1)(\alpha-1)}} z_e^{\frac{1}{(\beta-1)(\alpha-1)}}$$

$$l_{f,e} + \theta\bar{l} = \left(\left(\frac{w}{\alpha}\right)^{\frac{1}{\alpha-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{(\beta-1)(\alpha-1)}} z_e^{\frac{1}{(\beta-1)(\alpha-1)}}\right)^{\frac{(\beta-1)(\alpha-1)}{1-\alpha-\beta}}$$

Plugging $l_{f,e}$ into k_e and k_e into $l_{f,e}$, $l_{f,e}$ and k_e become:

$$l_{f,e} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} - \theta\bar{l} \tag{I.1}$$

$$k_e = \left(\frac{r}{\beta}\right)^{\frac{\alpha-1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} \tag{I.2}$$

Regular firm $l_{f,r}$ and k_r: Regular firm's production function is $y_r = z_r l_{f,r}^{\alpha} k_r^{\beta}$, $\alpha + \beta < 1$. Solving the maximization problem of the regular firm:

$$\max_{\{l_{f,r},k_r\}} \Pi_r = y_r - w l_{f,r}$$

$$\max_{\{l_{f,r},k_r\}} \Pi_r = z_r l_{f,r}^{\alpha} k_r^{\beta} - w l_{f,r} - r k_r$$

Taking the first-order condition w.r.t. $l_{f,r}$:

$$\frac{\partial \Pi_r}{\partial l_{f,r}} \Rightarrow \alpha z_r k_r^{\beta} \left(l_{f,r}\right)^{\alpha-1} - w = 0$$

Simplifying $l_{f,r}$ becomes:

$$l_{f,r} = \left(\frac{w}{\alpha z_r k_r^{\beta}}\right)^{\frac{1}{\alpha-1}}$$

Taking the first-order condition w.r.t. k_r:

$$\frac{\partial \Pi_r}{\partial k_r} \Rightarrow \beta z_r l_{f,r}^{\alpha} k_r^{\beta-1} - r = 0$$

Simplifying k_r becomes:

$$k_r = \left(\frac{r}{\beta z_r l_{f,r}^{\alpha}}\right)^{\frac{1}{\beta-1}}$$

Plugging $l_{f,r}$ into k_r and k_r into $l_{f,r}$, $l_{f,r}$ and k_r become:

$$l_{f,r} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \tag{I.3}$$

$$k_r = \left(\frac{r}{\beta}\right)^{\frac{\alpha-1}{1-\alpha-\beta}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \tag{I.4}$$

Total labor supply of free-laborers: Adding $l_{f,e}$ and $l_{f,r}$ and simplifying, the total labor supply of free-laborers become:

$$\bar{L} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right) - \theta\bar{l} \tag{I.5}$$

I.2 Equilibrium Wage Rate

Simplifying equation (I.5), we find the equilibrium wage rate:

$$w = \alpha \left(\frac{\bar{L} + \theta\bar{l}}{\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)} \right)^{\frac{1-\alpha-\beta}{\beta-1}} \tag{I.6}$$

Proof of Proposition E1. *The effect of capacity constraint on equilibrium wage rate is negative,* $\frac{\partial w}{\partial \bar{l}} < 0$*:*

$$\frac{\partial w}{\partial \bar{l}} = \frac{\theta(-\alpha-\beta+1)}{\beta-1} (\bar{L}+\theta\bar{l})^{\frac{1-\alpha-\beta}{\beta-1}-1} \frac{\alpha}{\left(\frac{\beta}{r}\right)^{\frac{\beta}{\beta-1}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{\beta-1}}} \tag{I.7}$$

When $\bar{l}$ is large, convict labor productivity and capacity become strategic complements. Thus, higher convict productivity further lowers equilibrium wages, $\frac{\partial^2 w}{\partial \bar{l} \partial \theta} > 0$.

$$\begin{aligned}\frac{\partial^2 w}{\partial \bar{l}\partial\theta} =& \frac{1-\alpha-\beta}{\beta-1}(\bar{L}+\theta\bar{l})^{\frac{1-\alpha-\beta}{\beta-1}-1}\left(\left(\frac{1-\alpha-\beta}{\beta-1}-1\right)\frac{\theta\bar{l}}{\theta\bar{l}+\bar{L}}+1\right) \\ & \frac{\alpha}{\left(\frac{\beta}{r}\right)^{\frac{\beta}{\beta-1}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{\beta-1}}}\end{aligned} \tag{I.8}$$

For equation (I.8) to be positive, it must be that $\left|\left(\frac{1-\alpha-\beta}{\beta-1}-1\right)\frac{\theta\bar{l}}{\theta\bar{l}+\bar{L}}\right| > 1$. *The first term consistently exceeds one, while the second remains below one. Thus, equilibrium wages decrease* *with increased convict productivity when $\bar{l}$ is sufficiently large, i.e., above the threshold $\bar{l}^*$ that uniquely solves* $\left|\left(\frac{1-\alpha-\beta}{\beta-1}-1\right)\right|\frac{\theta\bar{l}}{\theta\bar{l}+\bar{L}}=1$. ■

I.3 Labor Reallocation

To demonstrate the shift of labor from elite to regular firms, it is necessary to show that $\frac{\partial\frac{l_{f,e}}{l_{f,r}}}{\partial\bar{l}}<0$. This involves substituting the equilibrium wage into the expressions for $l_{f,e}$, and $l_{f,r}$:

$$\begin{aligned} l_{f,e} &= \left(\bar{L}+\theta\bar{l}\right)\left(z_e^{\frac{1}{1-\alpha-\beta}}+z_r^{\frac{1}{1-\alpha-\beta}}\right)^{-1} z_e^{\frac{1}{1-\alpha-\beta}}-\theta\bar{l} \\ l_{f,r} &= \left(\bar{L}+\theta\bar{l}\right)\left(z_e^{\frac{1}{1-\alpha-\beta}}+z_r^{\frac{1}{1-\alpha-\beta}}\right)^{-1} z_r^{\frac{1}{1-\alpha-\beta}} \end{aligned}$$

Then $\frac{l_{f,e}}{l_{f,r}}$ becomes:

$$\frac{l_{f,e}}{l_{f,r}}=\frac{z_e^{\frac{1}{1-\alpha-\beta}}}{z_r^{\frac{1}{1-\alpha-\beta}}}-\frac{\theta\bar{l}}{\bar{L}+\theta\bar{l}}\frac{z_e^{\frac{1}{1-\alpha-\beta}}+z_r^{\frac{1}{1-\alpha-\beta}}}{z_r^{\frac{1}{1-\alpha-\beta}}}$$

Proof of Proposition E2. *Taking the derivative of $\frac{l_{f,e}}{l_{f,r}}$ with respect to $\bar{l}$, we find the equilibrium level of labor demand ratio of both firms:*

$$\frac{\partial\frac{l_{f,e}}{l_{f,r}}}{\partial\bar{l}}=-\underbrace{\frac{\theta\bar{L}}{\left(\bar{L}+\theta\bar{l}\right)^2}}_{\downarrow\, in\ \bar{l}}\frac{z_e^{\frac{1}{1-\alpha-\beta}}+z_r^{\frac{1}{1-\alpha-\beta}}}{z_e^{\frac{1}{1-\alpha-\beta}}}$$

Therefore, an increase in the capacity constraint of convict labor leads to a reallocation of free labor from elite to regular firms $\left(\frac{\partial\frac{l_{f,e}}{l_{f,r}}}{\partial\bar{l}}<0\right)$. ■

I.4 Equilibrium Profits of the Elite Firm

Proof of Proposition E3. *The profit of the elite firm is* $\Pi_e=z_e\left(\theta\bar{l}+l_{f,e}\right)^{\alpha}k_e^{\beta}-wl_{f,e}-rk_e$. *Solving for the equilibrium level, plugging $l_{f,e}$ in Π_e:*

$$\Pi_e=z_e^{\frac{1-\beta}{1-\alpha-\beta}}\left(\frac{w}{\alpha}\right)^{\frac{\alpha(\beta-1)}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\alpha\beta}{1-\alpha-\beta}}k_e^{\beta}-w\left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}z_e^{\frac{1}{1-\alpha-\beta}}+\theta\bar{l}w-rk_e$$

Plugging k_e in Π_e:

$$\Pi_e = \left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} - w\left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} + \theta\bar{l}w$$
$$- r\left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}}$$

Defining parts of the equation:

$$(1) \equiv \left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}}$$
$$(2) \equiv -w\left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}}$$
$$(3) \equiv \theta\bar{l}w$$
$$(4) \equiv -r\left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} z_e^{\frac{1}{1-\alpha-\beta}}$$

Plugging w in Π_e for all parts of the equation:

$$(1) \equiv \left(\bar{L}+\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}}$$
$$(2) \equiv -\alpha\left(\bar{L}+\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} z_e^{\frac{1}{1-\alpha-\beta}}$$
$$(3) \equiv \alpha\theta\bar{l}\left(\bar{L}+\theta\bar{l}\right)^{\frac{1-\alpha-\beta}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}}$$
$$(4) \equiv -r\left(\bar{L}+\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} z_e^{\frac{1}{1-\alpha-\beta}}$$

Simplifying $\Pi_e = (1) + (2) + (3) + (4)$ becomes:

$$\Pi_e = \left(\bar{L}+\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right)$$
$$+ \alpha\theta\bar{l}\left(\bar{L}+\theta\bar{l}\right)^{\frac{1-\alpha-\beta}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}} \quad \text{(I.9)}$$

The impact of capacity constraint on equilibrium profits of the elite firm is positive ($\frac{\partial \Pi_e}{\partial \bar{l}} > 0$). The first part of equation I.9 demonstrates an increasing trend with respect to $\bar{l}$. Rewriting the second part of equation I.9:

$$
\begin{aligned}
=&\alpha\theta\bar{l}\left(\bar{L}+\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}}\frac{1}{\left(\bar{L}+\theta\bar{l}\right)}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}}\left(z_e^{\frac{1}{1-\alpha-\beta}}+z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}}\\
=&\frac{\theta\bar{l}}{\left(\bar{L}+\theta\bar{l}\right)}\underbrace{\left(\bar{L}+\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}}}_{\uparrow\, in\; \bar{l}}\alpha\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}}\left(z_e^{\frac{1}{1-\alpha-\beta}}+z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}}
\end{aligned}
$$

To see that $\frac{\theta\bar{l}}{(\bar{L}+\theta\bar{l})}$ *is also increasing in* $\bar{l}$*, applying the product rule:*

$$
\underbrace{(\theta(\bar{L}+\theta\bar{l})-\theta\theta\bar{l})}_{=\theta\bar{L}>0\Rightarrow\uparrow\, in\; \bar{l}}\underbrace{\left(\frac{1}{(\bar{L}+\theta\bar{l})^2}\right)}_{\uparrow\, in\; \bar{l}}
$$

Therefore, it holds that $(\frac{\partial\Pi_e}{\partial\bar{l}}>0)$.

I.5 Equilibrium Profits of the Regular Firm

This section provides the proof of Proposition E4. The profit of the regular firm is $\Pi_r = z_r l_{f,r}^{\alpha}k_r^{\beta}-wl_{f,r}-rk_r$*. Solving for the equilibrium level, plugging* $l_{f,r}$ *in* Π_r*:*

$$
\Pi_r = z_r^{\frac{1-\beta}{1-\alpha-\beta}}\left(\frac{w}{\alpha}\right)^{\frac{\alpha(\beta-1)}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\alpha\beta}{1-\alpha-\beta}}k_r^{\beta}-w\left(\frac{w}{\alpha}\right)^{\frac{\beta-)}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}-rk_r
$$

Plugging k_r *in* Π_r*:*

$$
\begin{aligned}
\Pi_r =&\left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}-w\left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}\\
&-r\left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}
\end{aligned}
$$

Defining parts of the equation:

$$
\begin{aligned}
(1)&\equiv\left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}\\
(2)&\equiv -w\left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}\\
(3)&\equiv -r\left(\frac{w}{\alpha}\right)^{\frac{-\alpha}{1-\alpha-\beta}}\left(\frac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}}z_r^{\frac{1}{1-\alpha-\beta}}
\end{aligned}
$$

Plugging w in Π_r for all parts of the equation:

$$(1) \equiv \left(\bar{L} + \theta \bar{l}\right)^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}}$$
$$(2) \equiv -\alpha \left(\bar{L} + \theta \bar{l}\right)^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}}$$
$$(3) \equiv -r \left(\bar{L} + \theta \bar{l}\right)^{\frac{\alpha}{1-\beta}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}}$$

Simplifying $\Pi_r = (1) + (2) + (3)$ *becomes:*

$$\Pi_r = \left(\bar{L} + \theta \bar{l}\right)^{\frac{\alpha}{1-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right) \tag{I.10}$$

The impact of capacity constraint on equilibrium profits of the regular firm is positive $\left(\frac{\partial \Pi_r}{\partial \bar{l}} > 0\right)$*:*

$$\frac{\partial \Pi_r}{\partial \bar{l}} = \frac{\theta \alpha}{1-\beta} \left(\bar{L} + \theta \bar{l}\right)^{\frac{\alpha}{1-\beta}-1} z_r^{\frac{1}{1-\alpha-\beta}} \left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right). \ \blacksquare$$

I.6 Capital–Labor Cost Ratio

The capital-to-labor cost ratio for the region is denoted as $CI \equiv \frac{rk_e + rk_r}{w\bar{L} + w_c \bar{l}}$. Since $w_c = 0$, the ratio simplifies to:

$$CI \equiv \frac{rk_e + rk_r}{w\bar{L}}.$$

Proof of Proposition E4. *The capital-to-labor cost ratio rises with the convict labor capacity constraint, i.e.,* $\frac{\partial CI}{\partial \bar{l}} > 0$*. We deduce this from the fact that as* $\bar{l}$ *increases, both* k_e *and* k_r *increase and* w *decreases.* $\blacksquare$

I.7 Union Membership and Strikes

Benefit of workers from striking: Should the strike succeed, the value or benefit (Ω) accrued by the employee is:

$$\Omega = \left(w(l_c = 0) - w(l_c = \bar{l})\right) \bar{L} \tag{I.11}$$

Probability of workers joining a union and striking: Assuming a strike can be prevented with probability $1 - p$ or succeed with p, with success eliminating convict labor use and failure causing a utility loss, δ. Free labor will strike if:

$$J < p\Omega - (1-p)\delta = p\left((w(l_c = 0) - w(l_c = \bar{l})\right)\bar{L} - (1-p)\delta \tag{I.12}$$

Proof of Proposition E5. *The RHS of equation (I.12) increases in correlation with $\bar{l}$:*

$$J < p\underbrace{\left(\underbrace{w(l_c = 0)}_{\frac{\partial w(l_c=0)}{\partial \bar{l}}=0} - \underbrace{w(l_c = \bar{l})}_{\frac{\partial w(l_c=\bar{l})}{\partial \bar{l}}<0}\right)}_{\uparrow\, in\, \bar{l}}\bar{L} - (1-p)\delta$$

To see that $\left(w(l_c = 0) - w(l_c = \bar{l})\right)$ is increasing in $\bar{l}$, let's first plug in $w(l_c = 0)$ and $w(l_c = \bar{l})$:

$$w(l_c = 0) - w(l_c = \bar{l}) = \alpha\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}}\left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}}\left(\bar{L}^{\frac{1-\alpha-\beta}{\beta-1}} - (\bar{L}+\theta\bar{l})^{\frac{1-\alpha-\beta}{\beta-1}}\right)$$

Taking the derivative of this expression w.r.t. $\bar{l}$, we find:

$$\frac{\partial w(l_c = 0) - w(l_c = \bar{l})}{\partial \bar{l}} = -\frac{1-\alpha-\beta}{\beta-1}\theta\alpha\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}}\left(z_e^{\frac{1}{1-\alpha-\beta}} + z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}}(\bar{L}+\theta\bar{l})^{\frac{1-\alpha-\beta}{\beta-1}-1} > 0$$

Therefore, an increase in convict labor capacity results in increased levels of union membership and strike occurrences. ■

J Further Quantitative Analysis

J.1 Calibration

The calibration exercise evaluates the impact of convict labor on labor market dynamics and assesses a scenario where unions eliminate convict labor use. The model aligns with moments observed in the data on average wage and capital-labor cost ratio when calibrated.

The parameters are α, β, θ, r, $\bar{l}$, $\bar{L}$, and $z_e = z_r = z$, as productivity differences between elite and regular firms are not discernible. Convict labor productivity θ is sourced from U.S. Bureau of Labor (1925), comparing convict and free labor output. The capital rental rate r is set at 0.0919, per Bodenhorn and Rockoff (1992).[2] Free laborers are normalized to 1, and convict laborers match the convict-to-free labor ratio $\frac{\bar{l}}{\bar{L}}$ from Section 3 data.

Table J1: Calibration

Parameter	Description	Value
α	Share of labor in production	0.194
β	Share of capital in production	0.766
θ	Productivity of convict labor	0.7
r	Rental rate of capital	0.0919
$\bar{l}$	Convict laborers	0.16
$\bar{L}$	Free laborers	1
$z_e = z_r = z$	Firm productivity	1

Note: The model's parameters include α, β, θ, r, $\bar{l}$, $\bar{L}$, and $z_e = z_r = z = 1$. I sourced convict labor productivity, θ, from U.S. Bureau of Labor (1925). I set the capital rental rate, r, at 0.05 based on Bodenhorn and Rockoff (1992), normalized free laborers to 1, and aligned convict labor quantities to the observed convict-to-free labor ratio. I use α and β to target the two moments observed in the data.

Calibration Strategy and Model-Data Match. I target two key moments in the data: the drop in free labor wages and the rise in the capital-labor cost ratio due to increased convict labor in exposed counties. To precisely identify the model with three parameters and two target moments, I fix $z = 1$. The labor and capital shares, α and β, are set at 0.194 and 0.766, respectively, to align the model with these targets:

$$m(\lambda) = \begin{bmatrix} \frac{w(\alpha,\beta,l_c=\bar{l})}{w(\alpha,\beta,l_c=0)} - \frac{w^{exposed}}{w^{overall}} \\ \\ \frac{CI(\alpha,\beta,l_c=\bar{l})}{CI(\alpha,\beta,l_c=0)} - \frac{CI^{exposed}}{CI^{overall}} \end{bmatrix}$$

[2] Using Table 5.9 from Bodenhorn and Rockoff (1992), I average postbellum interest rates in the Southern U.S. from 1870-1991 and 1891-1904, aligning with Section 5.2.

The objective function can be expressed as follows, with W denoting the identity matrix:

$$\min_{\lambda} \left(m(\lambda)' W^{-1} m(\lambda) \right) \tag{J.1}$$

To identify the target moments, I used average wage and capital-labor cost ratio data from Haines et al. (2005) for all counties in 1870 and calculated their average growth rates. I then applied the causal estimates from Section 5.2 to determine changes in these variables in counties affected by the introduction of convict labor.

Table J1 lists the model parameters and their values used in the calibration. The first two rows of Table J2 show the model's performance in matching the target moments, demonstrating its ability to represent convict labor's impact on wages and capital-labor cost ratio, crucial for my empirical analysis in Section 5.2. Later rows show that convict labor increases regular firms' labor intensity, elite firms' capital intensity, and profits for both.

Table J2: Model-Data Match

Variable	**Model**	**Data**
$\frac{w_{exposed}}{w_{overall}}$	0.982	0.982
$\frac{CI_{exposed}}{CI_{overall}}$	1.11	1.04
$\frac{\Pi^e_{exposed}}{\Pi^e_{overall}}$	1.35	–
$\frac{\Pi^r_{exposed}}{\Pi^r_{overall}}$	1.07	–
$\frac{\left(\frac{k_e}{l_{f,e}}\right)_{exposed}}{\left(\frac{k_e}{l_{f,e}}\right)_{overall}}$	1.23	–
$\frac{\left(\frac{k_r}{l_{f,r}}\right)_{exposed}}{\left(\frac{k_r}{l_{f,r}}\right)_{overall}}$	0.98	–

Note: The table displays ratios comparing the post-introduction convict labor effects in exposed counties to all pre-exposure period. Drawing from Haines et al. (2005), I determined 1870 averages and growth rates, evaluated convict labor's impact with Section 5.2's coefficients, and estimated absent variables using calibrated parameters Table J1.

The model also aligns with untargeted moments: it predicts an average wage of \$221, close to the pre-convict labor wage of \$218 described in Section 3, and it estimates a labor share α of 0.194, matching the empirical figure of 0.18. This validates the model and confirms its consistency. Figure J1 shows key model variables' dynamics with varying convict labor levels, highlighting that convict labor reduces wages, increases the county's capital-labor cost ratio, and raises profits for both firm types, especially elite firms.

Table J3: Effect on Manufacturing Outcomes 1880-1900

	Dependent variable:				
	Avg. Wage Growth	Employment Share	Capital Cost Share Change	Output Value	Gross Profit
Panel A: Pre-Convict Labor Prison Dummy IV	(1)	(2)	(3)	(4)	(5)
Convict Labor per 100,000 (IHS)	-1.581^{*}	-1.920^{***}	0.448^{**}	$181{,}093^{**}$	$31{,}254^{**}$
	(0.816)	(0.670)	(0.204)	(77,583)	(13,983)
KP F-stat	86.79	86.05	86	86.03	86.03
Panel B: Pre-Convict Labor Prison Capacities IV					
Convict Labor per 100,000 (IHS)	-1.382^{*}	-1.914^{***}	0.451^{**}	$185{,}888^{**}$	$31{,}160^{**}$
	(0.771)	(0.629)	(0.204)	(88,966)	(15,330)
KP F-stat	121.15	120.26	120.24	120.4	120.4
Controls	×	×	×	×	×
State × Year FE	×	×	×	×	×
Mean Outcome	26.4	94.8	12.58	970,522	79,779
Clusters	1,154	1,231	1,237	1,298	1,298
Observations	2,265	2,439	2,463	2,526	2,526

Note: Each column reports coefficients and standard errors (in parentheses) from the regression displayed in equation 11. The observation unit is the county (1880), with convict labor presence (convicts per 100,000 inhabitants - IHS) as the independent variable. It is instrumented by pre-convict labor prisons (Panel A) and their capacities (Panel B). Outcome variables are the average manufacturing wage growth, manufacturing employment share, the change in capital-labor cost-share, manufacturing output value, and gross profits. Economic controls encapsulate population density, enslaved individuals' share, and farmland statistics (output value per acre and improved acres). Geographic controls account for distance to the coast, elevation, transport access, and proximity to large urban centers. The vote share of the Democratic Party is the socio-political control. The model includes state × year fixed effects, and standard errors are county-level clustered.

J.2 Further Details on the Calibration Strategy

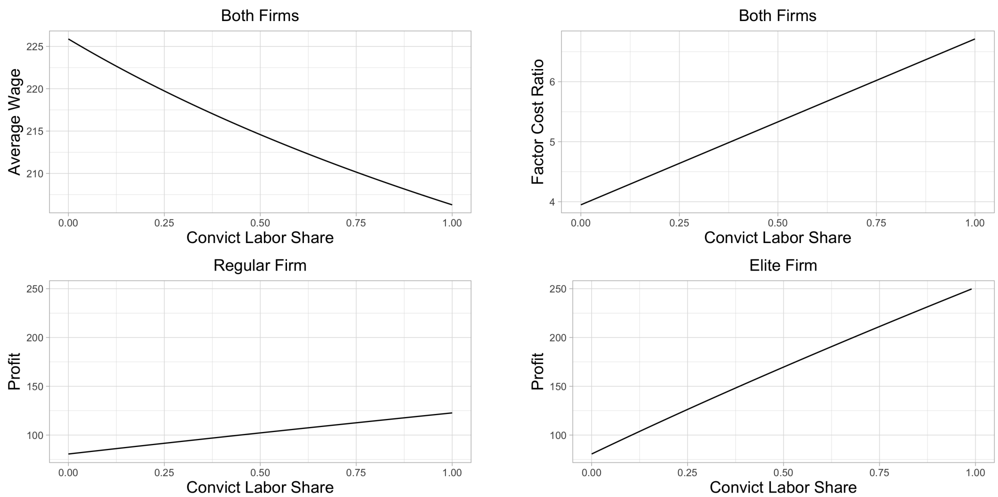
To identify the target moments, I calculated the average growth rates of wages and capital-labor cost ratios for all counties in 1870 using data from Haines et al. (2005). Using causal coefficient estimates from Section 5.2, I determined changes in these variables for counties affected by convict labor. Specifically, I used Table J3's estimates to calculate the ratios $\frac{w_{exposed}}{w_{overall}} = 0.982$ and $\frac{CI_{exposed}}{CI_{overall}} = 1.04$. To achieve the decline in wage and capital-labor cost ratios, I followed these steps:

Establishing baseline levels: The initial step is to determine the baseline levels for wages and capital-labor cost ratio before the introduction of convict labor. These are sourced from the county Census (Haines et al., 2005).

Calculating overall growth rates: After establishing the baseline levels, I calculate the overall growth rates in wages and capital-labor cost ratio. These growth rates reflect the general trends in wages and capital-labor cost ratio before the introduction of convict labor, as derived from the county Census (Haines et al., 2005). Then, I apply these growth rates to the initial baseline wage and capital-labor cost ratio to find the wage and capital-labor cost ratio with growth rates, denoted as $w_{overall}$ and $CI_{overall}$.

Adjusting for convict labor impact: Next, I apply the causal coefficient estimates of the decline in the wage and capital-labor cost ratio growth from the Supplementary Appendix Table J3 to the growth rates before the introduction of convict labor. This involves adjusting the baseline growth rates to account for the impact of convict labor. The adjusted rates represent the change in wage growth and capital-labor cost ratio growth due to the introduction of convict labor. Applying these growth rates to the baseline wages and capital-labor cost ratio results in $w_{exposed}$ and $CI_{exposed}$.

Figure J1. Calibrated Metrics: Average Wage, Factor Cost Ratio, and Profits



Note: The figure illustrates the variations in per-worker wage, capital-labor cost ratio for both firms, physical capital intensity for elite and regular firms, and profits for both firms as the capacity constraint of convict labor shifts, based on equations H.8, H.11, H.9, H.10.

Deriving ratios: Finally, I calculate the ratios $\frac{w_{exposed}}{w_{overall}} = 0.982$ and $\frac{CI_{exposed}}{CI_{overall}} = 1.04$. I jointly optimize the loss function for both ratios in calibrating the model.

K Model Extension: Firm Type Distribution

In this section, I extend the model to allow for a general firm-type distribution for elite and regular firms: η to capture elite and $1-\eta$ to capture regular and study the implications of η on $\partial w/\partial \bar{l}$. The equilibrium labor demand $l_{f,e}$ and $l_{f,r}$ remain the same as Section H. Combining $l_{f,e}$ and $l_{f,r}$, and firm weights η and $(1-\eta)$ to find the total free-labor demand and equating it to the total labor supply of free-laborers, $\bar{L}$, characterizes the general equilibrium of the model:

$$\bar{L} = \left(\frac{w}{\alpha}\right)^{\frac{\beta-1}{1-\alpha-\beta}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\eta z_e^{\frac{1}{1-\alpha-\beta}} + (1-\eta) z_r^{\frac{1}{1-\alpha-\beta}}\right) - \eta\theta\bar{l} \tag{K.1}$$

Wage rate of free-laborers, w:

$$w = \alpha \left(\frac{\bar{L} + \eta\theta\bar{l}}{\left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} \left(\eta z_e^{\frac{1}{1-\alpha-\beta}} + (1-\eta) z_r^{\frac{1}{1-\alpha-\beta}}\right)} \right)^{\frac{1-\alpha-\beta}{\beta-1}} \tag{K.2}$$

The effect of capacity constraint on equilibrium wage rate is negative, $\frac{\partial w}{\partial \bar{l}} < 0$:

$$\frac{\partial w}{\partial \bar{l}} = \frac{\eta\theta(-\alpha-\beta+1)}{\beta-1} (\bar{L} + \eta\theta\bar{l})^{\frac{1-\alpha-\beta}{\beta-1}-1} \frac{\alpha}{\left(\frac{\beta}{r}\right)^{\frac{\beta}{\beta-1}} \left(\eta z_e^{\frac{1}{1-\alpha-\beta}} + (1-\eta) z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{\beta-1}}}$$

When $\eta = 1$—all firms are elite—convict labor negatively affects wages. But when $\eta = 0$—no elite firms—convict labor does not impact wages ($\frac{\partial w}{\partial \bar{l}} = 0$).

Profits of Elite and Regular Firms:

$$\begin{aligned} \Pi_e = {} & \left(\bar{L} + \eta\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} z_e^{\frac{1}{1-\alpha-\beta}} \left(\eta z_e^{\frac{1}{1-\alpha-\beta}} + (1-\eta) z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right) \\ & + \alpha\theta\bar{l} \left(\bar{L} + \eta\theta\bar{l}\right)^{\frac{1-\alpha-\beta}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{\beta}{1-\beta}} \left(\eta z_e^{\frac{1}{1-\alpha-\beta}} + (1-\eta) z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{1-\alpha-\beta}{1-\beta}} \end{aligned} \tag{K.3}$$

The impact of capacity constraint on equilibrium profits of elite firms is positive ($\frac{\partial \Pi_e}{\partial \bar{l}} > 0$).

$$\Pi_r = \left(\bar{L} + \eta\theta\bar{l}\right)^{\frac{\alpha}{1-\beta}} z_r^{\frac{1}{1-\alpha-\beta}} \left(\eta z_e^{\frac{1}{1-\alpha-\beta}} + (1-\eta) z_r^{\frac{1}{1-\alpha-\beta}}\right)^{\frac{\alpha}{\beta-1}} \left(\frac{\beta}{r}\right)^{\frac{1}{1-\beta}} \left(\frac{r}{\beta}(1-\alpha-\beta)\right) \tag{K.4}$$

The impact of capacity constraint on equilibrium profits of the regular firms is positive ($\frac{\partial \Pi_e}{\partial \bar{l}} > 0$).

Union Membership and Strikes: The union and strike aspect of the model is unchanged. As previously, expanding convict labor capacity leads to more union memberships and strikes.

L Constant Elasticity of Substitution

This model includes a representative firm that employs both convict and free labor. Convict labor is unpaid, while free labor is paid ($w_f = w > 0$). The firm has a capacity limit for convict labor, $l_{c,e} \leq \bar{l}$.

The production function and the maximization problem:

$$Y = \left(\eta \bar{l}^\rho + (1-\eta) l_f^\rho\right)^{\frac{\alpha}{\rho}} \tag{L.1}$$

$$\max \left(\eta \bar{l}^\rho + (1-\eta) l_f^\rho\right)^{\frac{\alpha}{\rho}} - w l_f$$

$$\frac{\partial}{\partial l_f} : \alpha \left(\eta \bar{l}^\rho + (1-\eta) l_f^\rho\right)^{\frac{\alpha}{\rho}-1} (1-\eta) l_f^{\rho-1} - w$$

Wage becomes:

$$w = \alpha \left(\eta \bar{l}^\rho + (1-\eta) l_f^\rho\right)^{\frac{\alpha-\rho}{\rho}} (1-\eta) l_f^{\rho-1} \tag{L.2}$$

Taking the logarithm:

$$\log w = \log(\alpha) + \frac{\alpha-\rho}{\rho} \log\left(\eta \bar{l}^\rho + (1-\eta) l_f^\rho\right) + \log(1-\eta) + (\rho-1)\log(l_f)$$

Differentiate w.r.t. $\bar{l}$:

$$\frac{\partial \log w}{\partial \bar{l}} = \frac{\eta(\alpha-\rho)\bar{l}^{\rho-1}}{\eta(\bar{l}^\rho - l_f^\rho) + l_f^\rho} \tag{L.3}$$

One can estimate the elasticity of substitution parameter, ρ, in the following way:

$$\log w = \log(\alpha) + \frac{\alpha-\rho}{\rho} \log\left(\eta \bar{l}^\rho + (1-\eta) l_f^\rho\right)$$
$$+ \log(1-\eta) + (\rho-1)\log(l_f)$$

$$\frac{\partial \log w}{\partial \log Y} = \left(\frac{\alpha-\rho}{\rho}\right) \tag{L.4}$$

Once $\frac{\partial \log w}{\partial \log Y}$ and α from the data are known, one can get a unique ρ from equation (L.4), since $\left(\frac{\alpha-\rho}{\rho}\right)$ is monotone in ρ. The estimation of ρ is carried out independently of η, simplifying the process as η is not necessary to determine ρ.

Calibration. First, I find $\alpha = 0.18$ by dividing the total wage by manufacturing output and averaging the years in the analysis (Haines et al., 2005). Then, I regress average wage growth on log(manufacturing output) and log(convict labor share), controlling for convict labor share since estimating the CES-parameter ρ depends on convict labor being present ($\bar{l} > 0$). Without convict labor ($\bar{l} = 0$), the CES parameter drops, resulting in a standard Lucas-span-of-control production with l_f^α.

The coefficient estimate β from the regression equation, although statistically insignificant, is -0.8, which is used in the left-hand side of equation (L.4). Plugging in these values:

$$-0.8 = \left(\frac{0.18 - \rho}{\rho}\right)$$

which gives ρ equal to 0.9.

The estimated value of ρ is close to 1, suggesting high substitutability between convict and free labor. However, this result should be interpreted cautiously due to data limitations. A ρ near 1 implies convict and free labor are nearly perfectly substitutable. The model in the main body is consistent with this estimation. Since ρ is greater than α, the model's predictions remain qualitatively the same as in the main text.

References

Bodenhorn, H. and Rockoff, H. (1992). Regional Interest Rates in Antebellum America. In *Strategic Factors in Nineteenth Century American Economic History*, pages 159–187. University of Chicago Press.

Haines, M. R. et al. (2005). *Historical, Demographic, Economic, and Social Data: the United States, 1790-2002*. Inter-university Consortium for Political and Social Research Ann Arbor, MI.

Hiller, E. T. (1914). Labor Unionism and Convict Labor. *Journal of Criminal Law and Criminology*, 5:851.

Hubmer, J. and Restrepo, P. (2022). Not a Typical Firm: Capital–Labor Substitution and Firms' Labor Shares. Working Paper.

U.S. Bureau of Labor (1886). *Second Annual Report of the Commissioner of Labor: Convict Labor.* Government Printing Office, Washington, DC.

U.S. Bureau of Labor (1925). *Convict Labor in 1923*. Government Printing Office, Washington, DC.